Motion of Telephone Wires in Wind

By D. A. QUARLES

This paper deals with the position of equilibrium of a loop of wire in a steady transverse wind and with the swinging of such a loop in one or more gusts of wind. In the first part, the loop is assumed to be inelastic and to swing as a rigid body. Under these conditions, nomograms are given from which may be read the deflection of loops of wire .104″ or .165″ in diameter as a function of steady wind velocity. The maximum additional swing of such a loop with a single gust and with a succession of gusts of given peak velocities may also be read from the nomograms. A chart is also included giving the effect of wind velocity on the sag of .104″ and .165″ hard drawn copper wires at tensions and span lengths common in the telephone plant.

UNTIL recent years, most of the important open wire toll circuits of the Bell System had the two wires of a pair spaced 12 inches apart. This wide spacing, with the consequent high mutual inductance between the several pairs on a pole line, limited the use of the lines for multiplex transmission with high frequency or "carrier" currents. A reduction in the separation of the wires of a pair with the retention of the present center-to-center spacing of the pairs was one of the measures which offered the opportunity of increasing the message carrying capacity of a pole line. The controlling factor in limiting such a reduction in spacing was the hazard of the wires of a pair swinging together in the wind thus interrupting or impairing the transmitted messages.

About two years ago the 12-inch spacing was reduced to 8 inches in some cases. This was considered to be as great a change as could be safely taken from a mechanical point of view, based on the available data. These data consisted in part of experiments made on an experimental line and in part of an analysis of the performance of certain working wires in the telephone plant which, for various reasons, had been installed on a close spaced basis.

It was realized that if the wires of pairs could be placed even closer together, materially lower crosstalk between the circuits would result, thus increasing the circuit capacity of open wire lines, and therefore effecting economies. Accordingly, a comprehensive investigation of the wire spacing problem was begun. As some of the factors involved in a theoretical determination of the chance of two parallel wires swinging together in the wind were rather obscure and difficult of evaluation, it was decided to attack the problem experimentally. A field site was selected some distance from New York, where the terrain and weather conditions were suitable for such an investigation, and an experimental station was constructed and appropriately equipped.

Some time will be required, however, before definite conclusions can be drawn from the experimental work of this new laboratory.

As an aid in the interpretation of the experimental results, certain theoretical work has been done on the dynamics of a wire loop swinging in the wind. It is this phase of the problem that is dealt with in this article.

In the first part of this discussion, the wire loop is treated as an inelastic, rigid body.[1] As it was later found that under the conditions applying in our problem there was a considerable increase in the sag of the wire due to the wind, an investigation was made of the magnitude of the correction required when the elasticity of the wire is taken into account, the results of which are given in the latter part of this article.

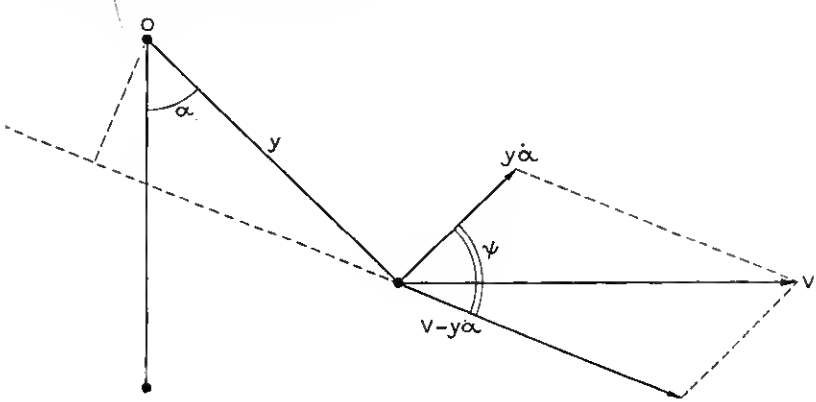

Fig. 1.

Consider an element of the wire, shown in Fig. 1 in cross-section, swinging about axis O, at a radius y. The wind is assumed horizontal and transverse to the axis. The sag a is also assumed small compared with the span length so that to a sufficient approximation the length of the wire is equal to the length of the span and the surface of the wire opposing the wind is independent of the angle of deflection (α) of the wire in the wind.

The velocity of the element of wire relative to axes fixed with respect to the earth is $y\dot{\alpha}$. The wind velocity relative to the same coordinate

[1] An article entitled "The Behavior of Overhead Transmission Lines in High Winds" by Professor E. H. Lamb, which appeared in the October 1928 Journal of the Institution of Electrical Engineers, gives an analysis of the inelastic, rigid loop problem which has been followed in general outline in the present treatment. There is disagreement, however, with one of the fundamental assumptions upon which Professor Lamb's analysis is based and our formulæ are therefore generally at variance with those derived in his article.

Mr. R. L. Peek, Jr. of Bell Telephone Laboratories, working independently, arrived at results in agreement with those given in the present article.

system is V and the wind velocity relative to the wire at any instant is therefore the vector difference $V - y\dot{\alpha}$ which has the magnitude

$$\sqrt{V^2 + (y\dot{\alpha})^2 - 2Vy\dot{\alpha}\cos\alpha}.$$

It is assumed that the wind pressure against this element is proportional to the square of this vector and acts along its direction. The moment about the axis of the wind pressure on the element ds is therefore given by:

$$k[V^2 + (y\dot{\alpha})^2 - 2y\dot{\alpha}V\cos\alpha]y\cos\psi ds,$$

where k is the ratio of wind pressure per unit length to square of velocity. Evaluating $\cos\psi$ and noting that $y\dot{\alpha}$ is small compared with V, this reduces to:

$$kdsyV^2\cos\alpha\left[1 - \frac{y\dot{\alpha}}{V}\left(\cos\alpha + \frac{1}{\cos\alpha}\right)\right].$$

Putting

$$y = a\left(1 - \frac{X^2}{C^2}\right)$$

and

$$ds = dx$$

and integrating, the total moment of wind pressure is

$$\frac{4}{3}kV^2aC\cos\alpha - \frac{16}{15}kVa^2C\dot{\alpha}(\cos^2\alpha + 1).$$

If the line through the supports is inclined to the horizontal by angle γ this expression becomes:

$$\frac{4}{3}kV^2aC\cos\alpha\cos\gamma - \frac{16}{15}kVa^2C\dot{\alpha}(\cos^2\alpha + 1)\cos^2\gamma$$

The dynamic equation for the motion of the loop then becomes:

$$\ddot{\alpha} + \frac{kV}{m}(1 + \cos^2\alpha)\dot{\alpha} + \frac{5g}{4a}\sin\alpha = \frac{5kV^2\cos\alpha}{4ma\cos\gamma},$$

where m is the mass of unit length of wire.

Static equilibrium is then given by:

$$\tan\alpha = \frac{kV^2}{mg\cos\gamma}.$$

Proceeding with the analysis, an equation is found for small motions

about any position of equilibrium (deflection α) of the form

$$\ddot{\varphi} + 2\epsilon\dot{\varphi} + n^2\varphi = 0,$$

where

$$\epsilon = \frac{(1 + \cos^2 \alpha)\, kV}{2m},$$

and

$$n^2 = \frac{5g}{4a \cos \alpha}.$$

For cases of practical interest in this investigation $n^2 > \epsilon^2$ and the motion about equilibrium is periodic and of period

$$T = \frac{2\pi}{\sqrt{n^2 - \epsilon^2}} = 2\pi \sqrt{\frac{4a \cos \alpha}{5g}}.$$

where a is the sag in feet and g the acceleration of gravity in feet per second per second. The ratio of the period of small oscillations about equilibrium to the period when α is zero is given by $T/T_0 = \sqrt{\cos \alpha}$.

The damping as measured by the ratio of successive half swings, λ, is given by

$$\log_e \lambda = \frac{\pi\epsilon}{\sqrt{n^2 - \epsilon^2}} \doteq \frac{\pi}{n}\epsilon$$

If a wire, held at a deflection α by a steady wind V, is subjected to a gust of wind having maximum velocity V_1, the additional throw of the wire will depend on the duration of the gust and may in general be either greater than or less than the increase in steady deflection which V_1, if sustained, would produce. The maximum throw will be given by a gust of most favorable duration and μ_{ms} has been defined as the ratio of this maximum throw to the increase in deflection that would result if the peak velocity were sustained. Similarly, for a periodic succession of gusts, there is a most favorable timing which in general will produce displacements greater than would a wind which sustained the velocity of the gust peaks. The ratio of the throws produced by a most favorably timed succession of gusts to the increase in deflection which would result if the peak velocity of the gusts were sustained, has been defined as μ_{mp}.

The formulæ derived above have been applied to the practical conditions of the telephone line problem,[2] where our interest is centered in hard drawn copper wire, commonly of .104″ or .165″ diameter, with spans ordinarily from 90 to 200 feet and sags commonly from 7″ to 20″

[2] This work was carried out in the Bell Telephone Laboratories by Mr. V. Nekrassoff.

though occasionally considerably greater. The method, which will be described in more detail elsewhere, was to reduce the expressions for wind pressure per unit length of wire, F, angular displacement α, periods of small oscillations, T and T_0, damping constant λ, and the effects of single and periodic gusts, μ_{ms} and μ_{mp}, to explicit functions of the wind velocity in miles per hour, the diameter of the wire in inches, sag of the wire in inches and trigonometric functions of the deflection of the loop α and inclination of the loop γ. The factor k does not appear directly in the equations, having been replaced by fractional powers of wind velocity and wire diameter derived from the experimental results of Relf.[3]

The following nomograms have been constructed by this method. Nomogram No. 1 (Fig. 2) gives the steady deflection α of a span of wire inclined to the horizontal at an angle γ and the force in pounds per linear foot of wire for a normal wind of velocity V. It also gives the ratio of the period of small oscillations about the equilibrium position to the natural period about the vertical position, this ratio depending only on α. The actual value of the period in seconds may be read on nomogram No. 2 (Fig. 3).

By the use of nomogram No. 3 (Fig. 4), the damping constant λ, and the gust ratios μ_{ms} and μ_{mp} may be computed from the sag a, the wind velocity V and the diameter D.

These nomograms in short give the numerical solution for our problem for wires of the two diameters assumed, namely .104″ and .165″.

Two major assumptions should be noted, first, that the wire loop swings in a plane and second, that the wire is inelastic. The first assumption has a certain justification in that each element of wire if independent of adjacent elements would be in equilibrium in the same deflected angle α as is found for the loop as a whole. Expressing this in another way—if it be assumed that the wind is uniform along the span there would be no forces, considering only first order effects, to distort the loop out of a plane.

The second assumption is not so readily justified, in fact the sag of the wire may be greatly affected by the wind pressure. The equilibrium deflection α is, however, independent of the sag of the wire and is found to be the same when the elasticity of the wire is taken into account as that derived for an inelastic wire.

Considering only the case where the line through the supports is horizontal ($\gamma = 0$), we define $2r$ as the unstressed length of wire in the loop and note that this may be either greater or less than the span length $2c$ depending upon the tension at which the wire is suspended.

[3] British Advisory Committee for Aeronautics—Report No. 102.

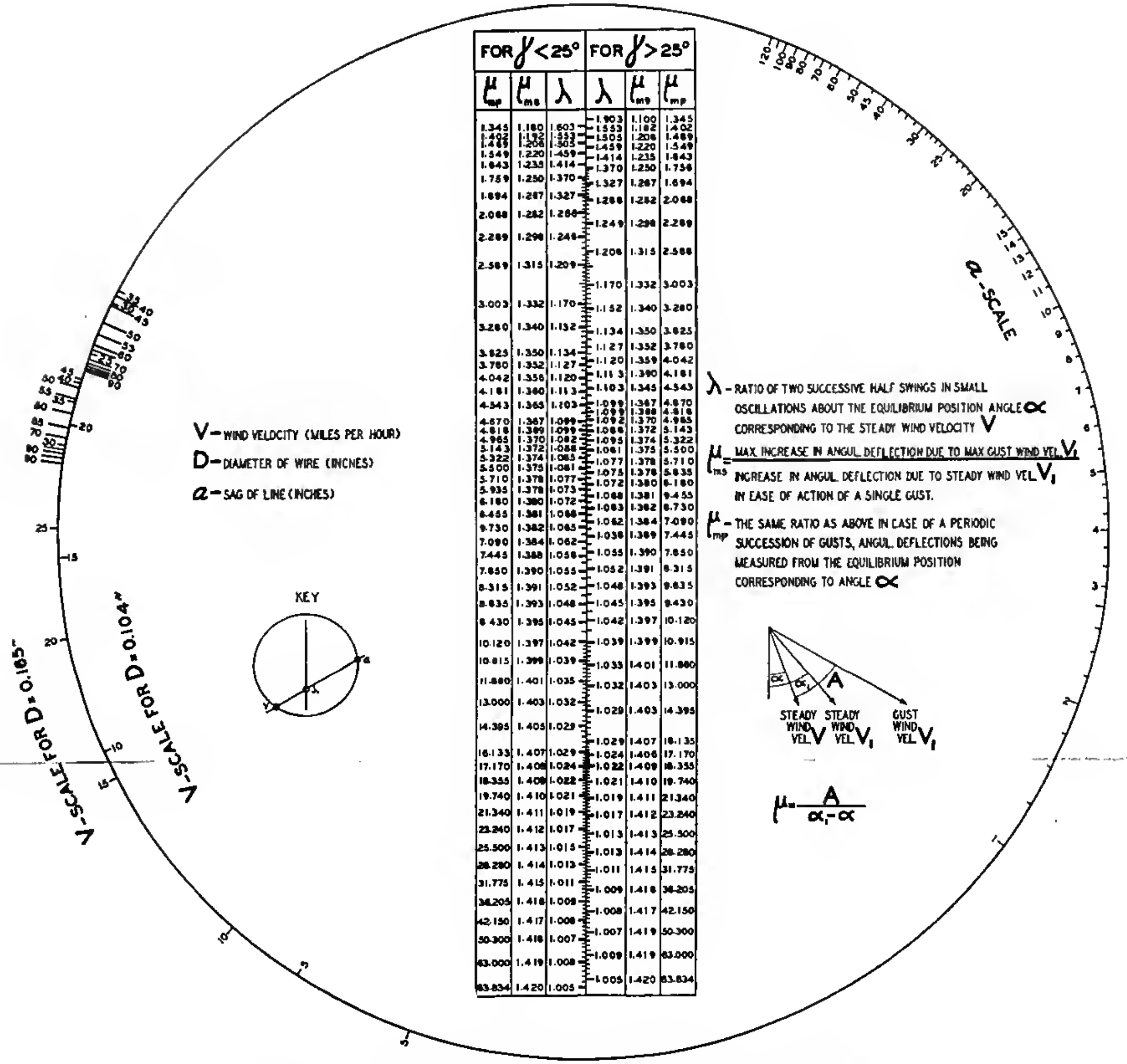

Fig. 4—Chart 3. Circular Alinement Nomogram Representing Resulting Effects of Periodic Gusts of Wind Superimposed on a Steady Wind.

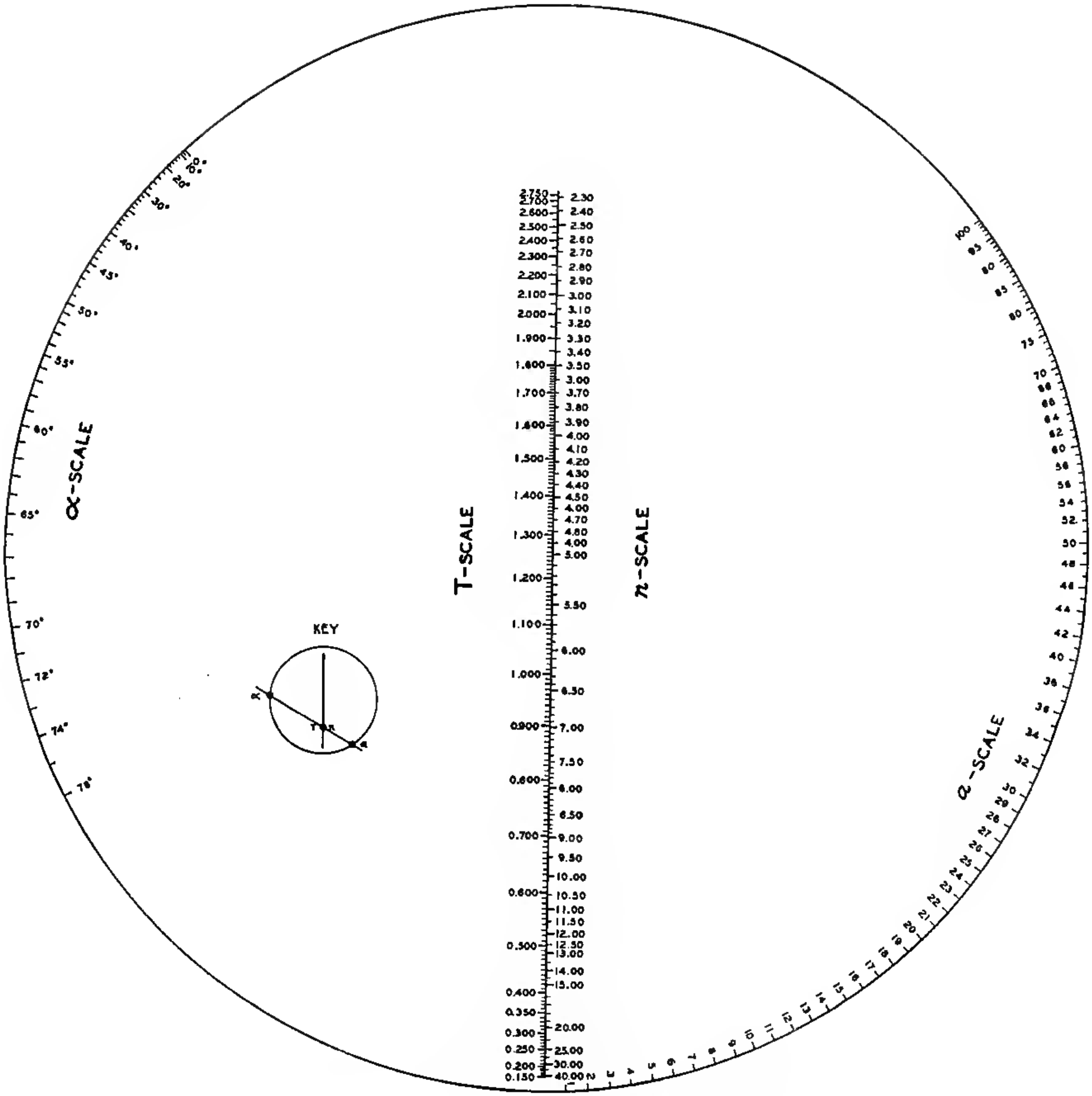

Fig. 3—Chart 2. Steady Wind Circular Alinement Nomogram Representing the Solution of

$$T^2 = \frac{4\pi^2}{15g} a \cos \alpha,$$

where

$T = \frac{2\pi}{n}$ = Period of Small Oscillations of Wire Loop (in Seconds) about its Equilibrium Position.

α = An Angle Corresponding to Equilibrium Position of Wire Under the Influence of Steady Wind Velocity V (see Nomogram No. 1).

a = Sag of Wire (in Inches).

g = Gravity Acceleration (32.16 Feet per Sec^2.).

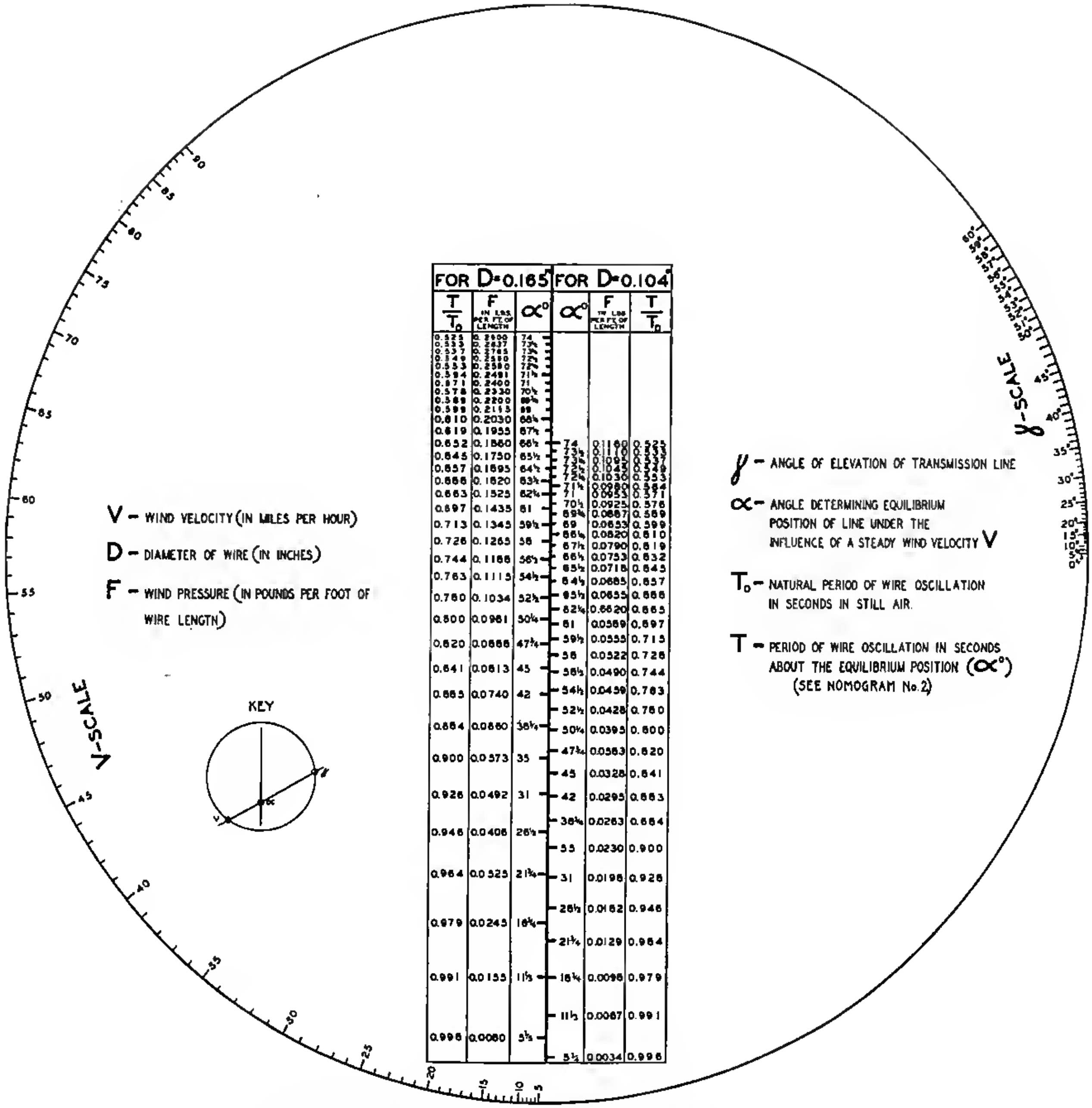

Fig. 2—Chart 1. Steady Wind Circular Alinement Nomogram for F, T/T_0 and $\alpha°$ Representing the Solution of Equation

$$tg\alpha = \frac{0.00422\,V^{2.0999}}{(100D)^{0.9154}\cos\gamma}.$$

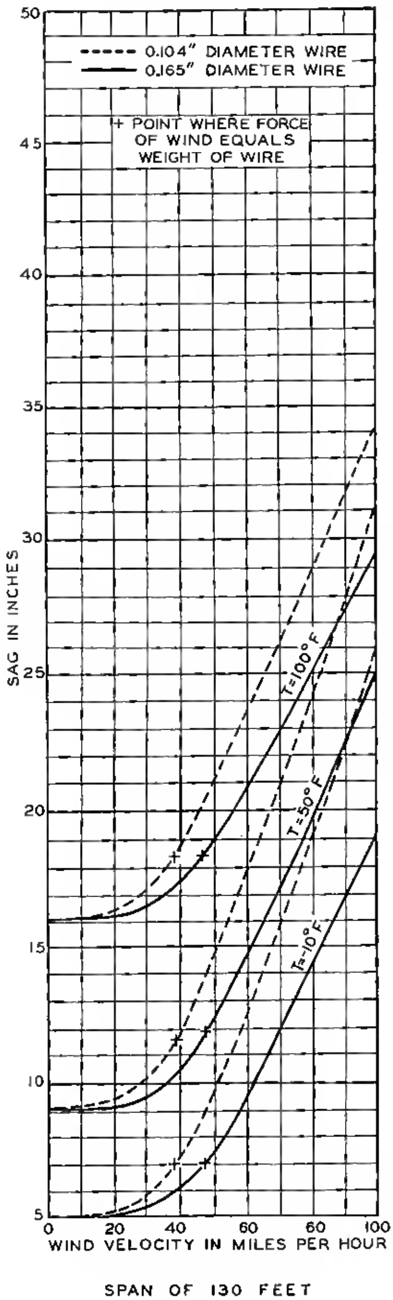
0.104" DIAMETER WIRE
0.165" DIAMETER WIRE
+ POINT WHERE FORCE OF WIND EQUALS WEIGHT OF WIRE
T=100° F
T=50° F
T=-10° F
SAG IN INCHES
WIND VELOCITY IN MILES PER HOUR
SPAN OF 130 FEET

Fig. 5-*A*.

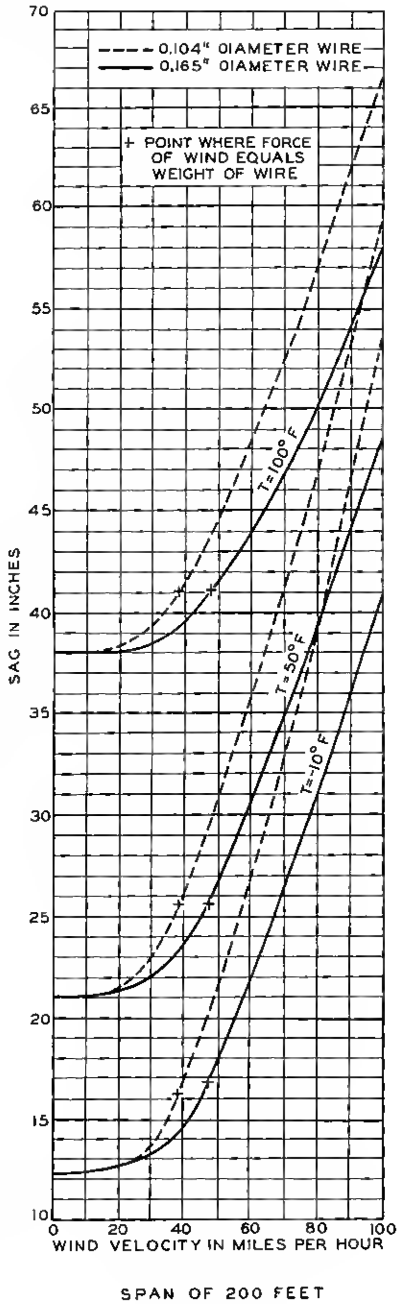
0.104" DIAMETER WIRE
0.165" DIAMETER WIRE
+ POINT WHERE FORCE OF WIND EQUALS WEIGHT OF WIRE
T=100° F
T=50° F
T=-10° F
SAG IN INCHES
WIND VELOCITY IN MILES PER HOUR
SPAN OF 200 FEET

Fig. 5-*B*.

If E is the modulus of elasticity in pounds per square inch cross-section, D the diameter in inches, a the sag in feet and m the weight of wire per linear foot, the approximate relationship [4] is:

$$a^3 + \frac{3c}{2}(c - r)a = \frac{3mc^4}{\pi D^2 E}.$$

As only horizontal winds normal to the line of supports are being considered, the wind pressure when the loop is in equilibrium is horizontal. The weight of the wire being vertical the two forces add at right angles, their resultant being the square root of the sum of their squares. This resultant lies of course in the plane of equilibrium of

Fig. 6—Test House and Line.

the loop. The wind pressure component is about equal to the gravity component for a velocity of 38 m.p.h. in the case of .104″ wire and about 47 m.p.h. in the case of .165″ wire. The effective weight of the wire under these conditions would be greater by a factor of $\sqrt{2}$ than the true weight. In general, m in the above formula is the effective weight of the wire per unit length.

A wire having a sag of 5″ in a 130′ span with a temperature of $-10°$ F. would have a sag of about 9″ at 50° F. and about 16″ at 100° F. due to thermal expansion. The sag of such a wire would be increased by wind pressure as shown in Fig. 5-*A*, the wind being given in true normal velocity. The figure shows the increase to be most marked for low temperatures and small diameters as would be expected. Similar

[4] Due to Mr. J. A. Carr of Bell Telephone Laboratories.

results are shown in Fig. 5-*B* for a span of 200′. Both indicate the marked increase of sag under not uncommon wind conditions.

While the above formula and charts give a fairly definite picture of the effect of elasticity on the solution of the problem of static equilibrium, the much more complex problem of the motion of an elastic loop in a varying wind has not been attacked.[5] The necessity for such additional refinements can probably not be determined until the field experiments above referred to have progressed to the point where fairly comprehensive data are available for analysis and for a check of the theoretical conclusions arrived at in this paper.

[5] An article by Karl Wolf in Zeitschrift für Angewandte Mathematik und Mechanik of April 1927 treats certain aspects of the dynamics of an elastic loop, with particular reference, however, to power lines. As yet, no attempt has been made to apply the results of this work to our particular problems.